Robust Fuzzy Segmentation of Magnetic Resonance Images

Dzung L. Pham
Laboratory of Personality and Cognition, Gerontology Research Center, NIA/NIH
5600 Nathan Shock Drive, Baltimore, MD 21224
dzung.pham@nih.gov

Abstract

A new approach for robust segmentation of magnetic resonance images is described. The approach is derived from a generalization of the objective function used in Pham and Prince's Adaptive Fuzzy C-means algorithm (AFCM). Within the objective function, an additional constraint is placed on the membership functions that forces them to be spatially smooth. Minimization of this objective function results in an unsupervised fuzzy segmentation algorithm that is robust to both intensity inhomogeniety artifacts as well as noise and other artifacts. The efficacy of the algorithm is demonstrated on simulated magnetic resonance images.

1. Introduction

Tissue classification is a necessary step in many medical imaging applications including the quantification of tissue volumes, the detection of pathology, and computer integrated surgery. Classification of voxels exclusively into distinct classes, however, is difficult because of artifacts such as noise and partial volume effects, which occur when multiple tissues contribute to a single voxel. To compensate for these artifacts, there has recently been growing interest in soft segmentation methods. In soft segmentations, voxels may be classified into multiple classes with a varying degree of membership. The fuzzy *C*-means clustering algorithm (FCM) is a soft segmentation method that has been used extensively for segmentation of magnetic resonance (MR) images because it is automatic and is applicable to a wide variety of problems [1]. Standard FCM, however, can not effectively compensate for intensity inhomogeneities, a common artifact in MR images.

Recently, Pham and Prince [5, 2] proposed an algorithm called the Adaptive Fuzzy C-means Algorithm (AFCM) that produces a soft segmentation while simultaneously adapting to intensity inhomogeneities in the image. The algorithm was derived by incorporating a gain field term into the objective function of the standard fuzzy c-means algorithm. Constraints on the gain field were used to ensure that the estimated field was smooth and slowly varying. While AFCM has been shown to be effective in correcting for inhomogeneities, its main disadvantage is that its performance degrades significantly with increased noise. One approach to solving this problem would be to preprocess the image with a smoothing filter. However, typical smoothing filters cause a loss of detail in the original image and are not mathematically consistent with the fuzzy clustering approach.

In this paper, we propose an enhancement to the AFCM algorithm to improve its robustness to noise. The improvement is formulated by directly injecting a new term into the objective function that constrains the behavior of the membership functions such that the membership value at each pixel depends not only on the data at that pixel, but also on the neighboring membership values. The resulting new algorithm is called the Fuzzy and Noise Tolerant Adaptive Segmentation Method (FANTASM).

2. Algorithm

The original AFCM algorithm is derived by minimizing the following objective function with respect to the membership functions, the centroids, and the gain field [2]:

$$J_{\text{AFCM}} = \sum_{j \in \Omega} \sum_{k=1}^{C} u_{jk}^{q} \|\mathbf{y}_j - g_j \mathbf{v}_k\|^2 + \lambda_1 \sum_{j \in \Omega} \sum_{r=1}^{R} (D_r * g)_j^2 + \lambda_2 \sum_{j \in \Omega} \sum_{r=1}^{R} \sum_{s=1}^{R} (D_r * D_s * g)_j^2 \qquad (1)$$

Here, Ω is the set of voxel locations in the image domain, q is a user-selected parameter that is constrained to be greater than one, u_{jk} is the membership value at voxel location j for class k, $\mathbf{y}_j$ is the observed (vector) image intensity at location j, $\mathbf{v}_k$ is the centroid of class k, and g_j is the scalar gain field . The total number of classes C is assumed to be known. The parameter q determines the amount of "fuzziness" of the resulting classification [1], and is typically set to 2. The last two terms are first and second order regularization terms used to constrain g_j to be spatially smooth and slowly varying. The parameters λ_1 and λ_2 control the amount of regularization and are determined empirically.

The AFCM objective function is minimized when high membership values are assigned where pixel intensities are close to the centroid for its particular class, and low membership values are assigned where the pixel data is far from the centroid. The gain field adapts to inhomogeneities by allowing the centroids to vary spatially through the image with the observed data. Optimization of the objective function is accomplished using an iterative algorithm [2] and implemented on a computer workstation. Note that the objective function (1) does not assume any spatial smoothness with respect to the membership functions. This deficiency causes AFCM to be sensitive to noise. To address this problem we propose a new objective function:

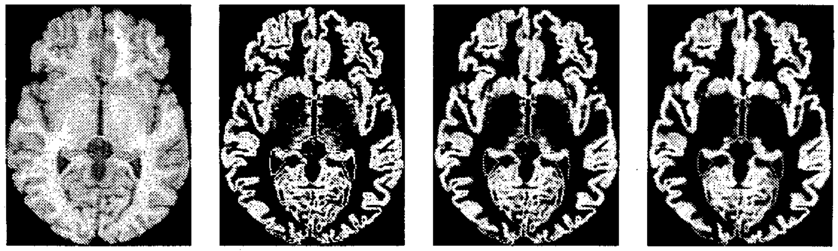
$$J_{\text{FANTASM}} = J_{\text{AFCM}} + \frac{\beta}{2} \sum_{j \in \Omega} \sum_{k=1}^{C} u_{jk}^{q} \sum_{l \in N_j} \sum_{m \neq k} u_{lm}^{q} \qquad (2)$$

In this equation, N_j is the set of first order neighbors of pixel j. The additional penalty term forces the membership values at each pixel to be dependent on its neighbors. It is small when the membership value for a particular class is large and the membership values for the other classes at neighboring pixels is small (and vice versa). The term resembles the prior probability models used in Markov random field methods [3, 6]. The major differences are that u_{jk} are continuously-valued variables and that the parameter q is used within the term to maintain the fuzzy characteristics of the membership function. The parameter β determines the amount of smoothness that is placed upon the membership functions and is determined empirically.

Equation (2) can be minimized by solving for the zero gradient condition with respect to each unknown variable and iterating through each of these conditions. Unlike other methods for incorporating spatial smoothness in membership functions (cf. [4]), an advantage of the formulation in (2) is that the resulting equations for minimizing the objective function are almost identical to those derived by Pham and Prince [2]. The only difference is in the equation for computing the membership values. The algorithm steps can be stated as follows:

FANTASM steps

1. Obtain initial estimates of the centroids, c_k.



(a) (b) (c) (d)

Figure 1: Application of FANTASM to MR image: (a) original image, (b) FCM gray matter membership, (c) FANTASM gray matter membership ($\beta = 100$), (d) FANTASM gray matter membership ($\beta = 250$)

2. Compute membership functions using (3).

$$u_{jk} = \frac{(\|\mathbf{y}_j - g_j \mathbf{v}_k\|^2 + \beta \sum_{l \in N_j} \sum_{m \neq k} u_{lm}^q)^{-1/(q-1)}}{\sum_{i=1}^{C} (\|\mathbf{y}_j - g_j \mathbf{v}_i\|^2 + \beta \sum_{l \in N_j} \sum_{m \neq i} u_{lm}^q)^{-1/(q-1)}} \tag{3}$$

3. Compute gain field by solving the spatially varying difference equation for g_j:

$$\sum_{k=1}^{C} u_{jk}^q \langle \mathbf{y}_j, \mathbf{v}_k \rangle = g_j \sum_{k=1}^{C} u_{jk}^q \langle \mathbf{v}_k, \mathbf{v}_k \rangle + \lambda_1 (H_1 * g)_j + \lambda_2 (H_2 * g)_j \tag{4}$$

where H_1 and H_2 are defined in [2].

4. Compute centroids:

$$\mathbf{v}_k = \frac{\sum_{j \in \Omega} u_{jk}^q \mathbf{y}_j}{\sum_{j \in \Omega} u_{jk}^q}, \; k = 1, \ldots, C \tag{5}$$

5. Go to step 2 and repeat until convergence.

Using (3), the membership values at each pixel depend not only on the observed intensity value at that pixel, but also on its neighboring membership values. The membership value is higher when the neighboring membership values of the other classes are low. For scalar image data, Step 1 can be performed automatically using a histogram smoothing algorithm described in [5]. Equation (4) can be solved efficiently using a multigrid algorithm [2]. Convergence can be considered to be obtained when the change in the objective function is less than some threshold, or when the change in the membership values is less than some threshold. The latter criterion is used in this paper.

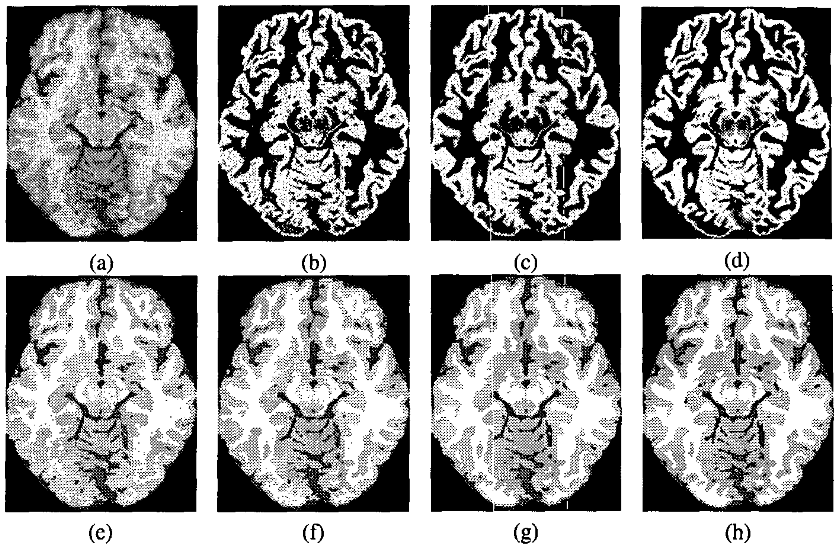


Figure 2: Simulated MR phantom results: (a) original image, (b) AFCM gray matter membership, (c) FCM gray matter membership, (d) true gray matter partial volume, (e) FCM maximum membership segmentation, (f) AFCM segmentation, (g) FANTASM segmentation, (h) true segmentation

3. Results

Figure 1 shows the results of applying FANTASM for different values of the β parameter. The algorithm was applied to a T1-weighted, 3-D MRI brain data set, thereby producing membership functions for gray matter, white matter, and cerebrospinal fluid. One slice from the data set and the resulting gray matter membership functions are shown. The standard FCM result shown in Figure 1b can be considered to be the special case where $\lambda_1 = 0$, $\lambda_2 = 0$, and $\beta = 0$. As β is increased, the membership function exhibits an increasingly smooth appearance. However, because a soft segmentation is produced, important details from the original image are still preserved even when β is set quite high. This is in contrast to Markov random field based methods that can cause soft segmentations to become nearly binary functions [6].

Figure 2 shows the results of applying various methods to simulated MR images taken from the Brainweb database [7]. A three-dimensional, T1-weighted image was simulated with 5% noise and 40% inhomogeneity. Figure 2a shows a single slice taken from the image volume. Figures 2b-c are a the corresponding slice taken from the gray matter membership functions computed by AFCM and FANTASM, respectively. Figure 2d shows the true partial volume fractions that were used in to create the simulated image. Although both AFCM and FANTASM produce reasonable results, the FANTASM result is clearly more robust to the noise in the image. In addition, the FANTASM result provides a fairly accurate estimate of the true partial volume fractions as well.

The improvements offered by FANTASM can also be seen when used as a hard segmentation

Table 1: Error measures from simulated MR results

Method	*Data set* 3%N,0%I	3%N,20%I	3%N,40%I	5%N,20%I	7%N,20%I
FANTASM-MCR	3.924%	4.160%	4.609%	5.209%	6.805%
AFCM-MCR	4.168%	4.322%	4.938%	6.765%	10.209%
FCM-MCR	3.988%	5.450%	9.046%	7.605%	10.515%
FANTASM-MSE	0.0183	0.0191	0.0211	0.0253	0.0363
AFCM-MSE	0.0210	0.0214	0.0244	0.0393	0.0659
FCM-MSE	0.0194	0.0272	0.0517	0.0432	0.0671

method. A hard segmentation can be produced from a fuzzy segmentation by assigning each pixel or voxel to the class of highest membership. This is called a maximum membership segmentation. In Figures 2e-f, both FCM and AFCM exhibit speckling because of the noise. The AFCM result, because it adapts to the inhomogeneities, is able to reconstruct some structures at the bottom and left side of the image that FCM does not. Figures 2g-h show that FANTASM provides a segmentation very close to the truth model. Tissue boundaries are smooth and connected, unlike the FCM and AFCM results.

Table 1 shows the errors resulting from applying the various algorithms to simulated MR images. T1-weighted, 3-D images were used. The columns show the errors for different levels of noise and inhomogeneity in the image. Noise was varied from 3% to 7% and inhomogeneity was varied from 0% to 40%. The rows show the errors obtained from applying the FANTASM, AFCM, and FCM algorithms, in terms of misclassification rate (MCR) [2] and mean-squared error (MSE) between the membership function and true partial volume of gray matter. For the 3% noise level, FANTASM performs only slightly better than AFCM. However, as the noise is increased, FANTASM yields a substantial improvement in segmentation accuracy. For all simulated MR results, the following parameter settings were used: $\lambda_1 = 2 \times 10^4, \lambda_2 = 2 \times 10^5$, and $\beta = 150$.

References

[1] J.C. Bezdek, L.O. Hall, and L.P. Clarke. Review of MR image segmentation techniques using pattern recognition. *Medical Physics*, 20:1033–1048, 1993.

[2] D.L. Pham and J.L. Prince. Adaptive fuzzy segmentation of magnetic resonance images. *IEEE Trans. on Medical Imaging*, 18(9):737–752, 1999.

[3] J. Zhang. The mean field theory in EM procedures for Markov random fields. *IEEE Trans. on Signal Processing*, 40(10):2570–2583, 1992.

[4] A.W.C. Liew, S.H. Leung, and W.H. Lau. Fuzzy image clustering incorporating spatial continuity. *IEE Proc.- Vis. Image Signal Process*, 147(2):185–192, 2000.

[5] D.L. Pham and J.L. Prince. An adaptive fuzzy c-means algorithm for image segmentation in the presence of intensity inhomogeneities. *Pattern Recognition Letters*, 20(1):57–68, 1999.

[6] K.V. Leemput, F. Maes, D. Vandermulen, and P. Seutens. Automated model-based tissue classification of MR images of the brain. *IEEE Trans. on Medical Imaging*, 18(10):897–908, 1999.

[7] D.L. Collins, A.P. Zijdenbos, V. Kollokian, J.G. Sled, N.J. Kabani, et al. Design and construction of a realistic digital brain phantom. *IEEE Trans. on Medical Imaging*, 17:463–468, 1998. http://www.bic.mni.mcgill.ca/brainweb.